

#### Introduction

Fundamentally, a Vector Network Analyzer (VNA) measures RF reflections. In a  $50\Omega$  system, reflections occur whenever an impedance other than 50 ohms is encountered by the RF signal. The reflection has a magnitude and a phase with respect to the magnitude and phase of the incident signal and may be characterized as a complex number. The VNA menu allows for a selection of charts which can display complex reflection coefficients in many different ways.

#### **Reflection and Reflection Coefficients**

In optics, if a beam of light traveling in a media with dielectric constant  $\varepsilon_1$  encounters a region with dielectric constant  $\varepsilon_2$ , there will be a reflection as shown in Figure 1.

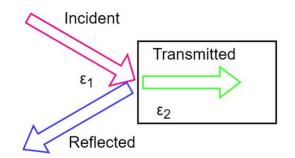


Figure 1

Figure 1 - Optical Reflection

The magnitude of the reflection is related to the difference between the two dielectric constants. The transmitted and reflected waves may be normalized by dividing by the incident wave to arrive at a *Reflection Coefficient*,  $\Gamma$ .

$$\Gamma_{Ref} = \frac{Reflected}{Incident}$$
  $\Gamma_{Trans} = \frac{Transmitted}{Incident}$ 

For instance, in the absence of loss, the coefficient for reflection might be 0.2, thus the coefficient for transmission must be 0.8. Here, the reflection coefficient is ratio-metric, and the magnitude of the incident signal is irrelevant.



Similarly, an RF voltage wave travelling on a  $50\Omega$  transmission line may encounter an impedance other than 50 ohms. This results in a reflection. Specifically, that reflection is given by:

Eq. 1)  $\Gamma = \frac{Z-50}{Z+50}$ 

Where Z is the encountered impedance.

If Z is a complex impedance –which can be expressed as R+jX– such as a resistor and capacitor in series, the reflection coefficient  $\Gamma$  will also be complex. We can choose to display the complex reflection coefficient in the following ways:

#### **Display Formats**

**Log Magnitude** – In this format, the magnitude of the complex reflection coefficient is calculated, the base 10 logarithm is applied, and the result multiplied by 20. The multiplication by 20 makes the result proportional to power instead of voltage.

 $LogMag = 20 * \log_{10}(|\Gamma|) = 20 * \log_{10}(\sqrt{a^2 + b^2})$ 

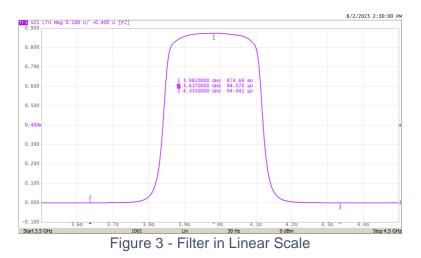
Where the complex reflection coefficient  $\Gamma$  is a+jb.

This format is convenient to examine a reflection coefficient over a very wide dynamic range. Details of the low loss in the passband of a filter along with a very deep stopband can be clearly seen in Figure 2. This would not be possible using a linear scale.

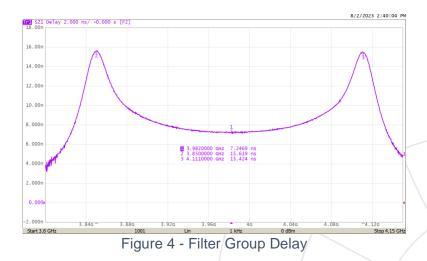




**Linear Magnitude** – The linear scale is convenient to examine a transmission coefficient such as S21, where the linear magnitude covers a range within two to three orders of magnitude. Figure 3 shows the same filter from Figure 2 but in the linear scale. Details at markers 2 and 3 are not distinguishable in this format.



**Delay** – The delay format shows the group delay, which is calculated from  $d\Phi/d\omega$  of S<sub>21</sub>.  $\Phi$  is the phase of the reflection coefficient and  $\omega$  is the radian frequency,  $2\pi f$ . The group delay of the filter in Figure 2 is shown below in Figure 4.

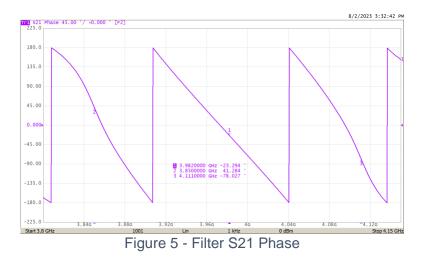


The group delay varies from 7.2 nS to 15.4 nS in the passband of the filter. If a modulated signal were passed through this filter such that some of the modulation bandwidth extends toward the peaks, there would be significant phase distortion (dispersion). Frequencies near the peaks might arrive at the receiver 5 nS or so later



than frequencies near the center. Filters can be designed with much flatter delay than this if needed, and this can be evaluated using this delay display mode.

**Phase** – the phase of the reflection coefficient can be displayed on the VNA screen. The phase is the arctangent of the ratio of the imaginary and real parts of the coefficient, but it represents the phase alignment of the reflected or transmitted signal with respect to the incident signal. If the two signals are in perfect alignment, the phase difference is zero. The phase depends greatly on the delay experienced by the signal. Figure 5 shows the phase of the signal passed through the filter of Figure 2.

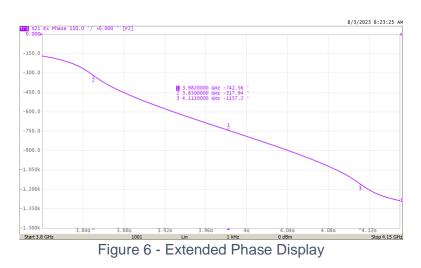


The phase wraps at -180 and 180 degrees. The curvature on the left and right sides is due to the delay variation shown in Figure *4*, whereas the phase is reasonably linear where the delay is flat in the middle of the passband.

**Extended Phase** – The phase is not wrapped in extended phase mode. The phase will be near zero at low frequencies and then become more and more negative at higher frequencies. The phase will change at a greater rate as more delay is experienced.

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**SWR** – The Standing Wave Ratio is used on reflection measurements such as S<sub>11</sub>. Reflections on a transmission line interfere with the incident wave, creating peaks and valleys in the voltage envelope of the RF wave. The ratio of the peaks to the valleys along the transmission line is the SWR. An SWR of 1:1 means there are no peaks or valleys, and hence no reflection. SWR may be calculated from the linear reflection coefficient,  $\Gamma$ .

 $SWR = \frac{\Gamma+1}{\Gamma-1}$ 

**Real and Imag** – The Real and Imaginary display formats depict the real or the imaginary components of a reflection coefficient. Note, these are not the real and imaginary components of an impedance. If there is no imaginary component, then the impedance contains no reactive component and must be resistive alone. If there is no real component, the impedance must be reactive alone.

**Polar** – The polar format plots the complex reflection coefficient within a unit circle with the real component on the horizontal axis and the imaginary component on the vertical axis. This is the most fundamental plot, as it displays the reflection coefficient in its most basic form. All other formats are useful derivations.

A reflection coefficient is essentially a complex vector. The polar format plots the magnitude and phase of this vector. A vector is often drawn as an arrow, but a dot at what would be the tip of the vector is drawn to reduce clutter when showing vectors for many frequencies.



A polar chart will have circles of magnitude and radial lines of angle, as shown below in Figure 7. Magnitude is shown in linear format here.

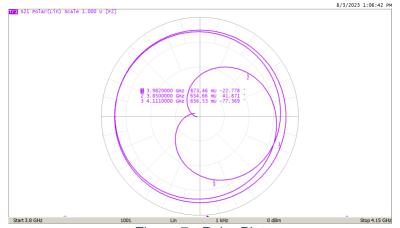


Figure 7 - Polar Plot

If Polar(log) is chosen, the magnitudes of the makers will be stated in dB format. The polar Smith formats, Smith(log), Smith(Lin), Smith(Re/Im), and Smith(R+jX) are Polar plots with a Smith chart overlay. The Smith overlay maps the polar reflection coefficient to the corresponding impedance, which gives rise to it as calculated from Eq. 1. The variations above merely change the values given by the markers.

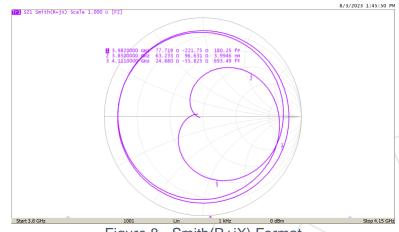


Figure 8 - Smith(R+jX) Format

Smith(log) will give the log magnitude and angle of the reflection. Smith(Lin) will give the linear magnitude and angle. Smith(Re/Im) will give the linear Cartesian format reflection. Smith(R+jX) will give the complex impedance at each point. Smith(G+jB) uses the



admittance overlay on the polar plot. Instead of an R+jX impedance, the marker value will show the G+jB admittance, which gives rise to the reflection, where the R+jX mode gives the impedance of a pair of series components, a resistance and a reactance. The equivalent inductance or capacitance at that frequency is also given. The G+jB admittance gives the conductance (inverse of resistance) and the susceptance of two components in parallel.

The Smith(R+jX) is convenient for analyzing an impedance, which is best modeled as a series impedance, while Smith(G+jB) is best for analyzing a parallel admittance.

#### Conclusion

Different display formats are useful for different measurements. This application note covers several different display formats and their uses. For example, the Log Mag format might be best for evaluating a filter, but the SWR will likely be used to evaluate the input match of an antenna, and the Smith(R+jX) is useful for measuring an impedance and perhaps working out a matching network.

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