METROLOGY AND SYSTEMATIC ERRORS

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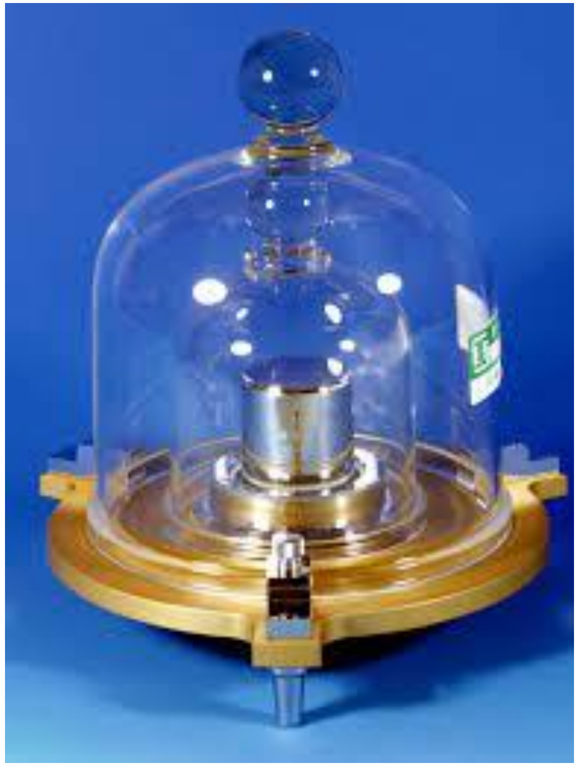
AGENDA

- Introduction to Metrology
- Uncertainties and Propagation of Uncertainties
- VNA Measurement Errors
- The 12-Term Error Model for 2-Port Measurement
- Errors and Residual Errors
- Estimation of Residual Errors
- Reflection and Transmission Measurement Uncertainty

WHAT IS METROLOGY?

- Metrology is the science of measurement
- This includes the definition of measurement units
- The SI (Système Internationale) defines seven such units.
    - Meter, Second, Ampere, Mole, Candela, Kelvin and Kilogram
    - All but Kilogram defined by physics
    - The Kilogram is a physical prototype!



The IPK, Kilogram Standard

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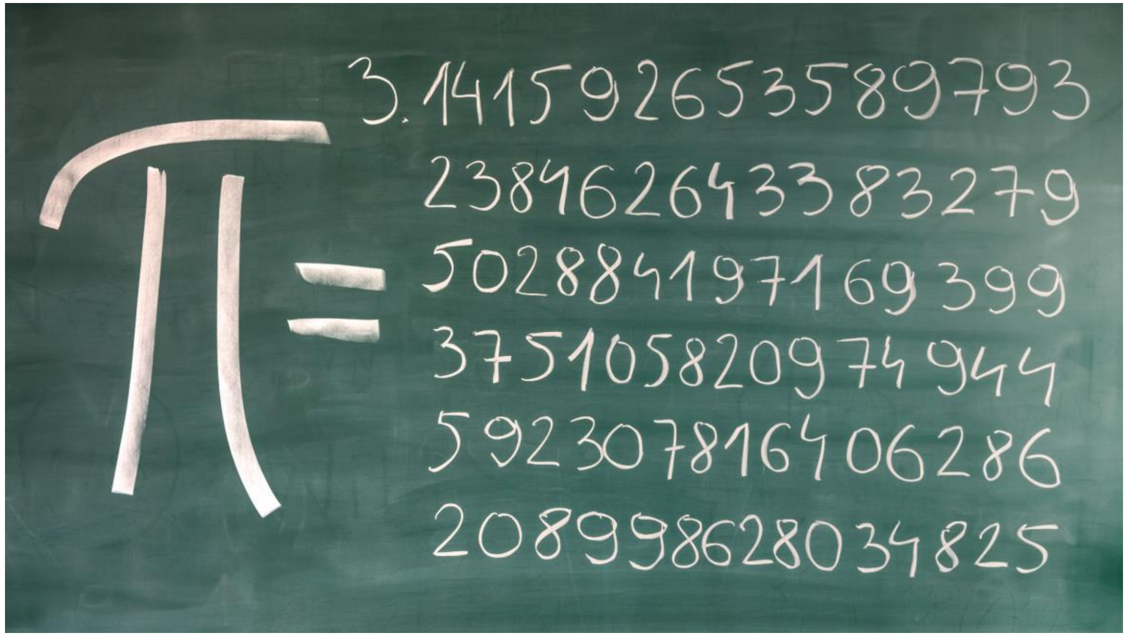
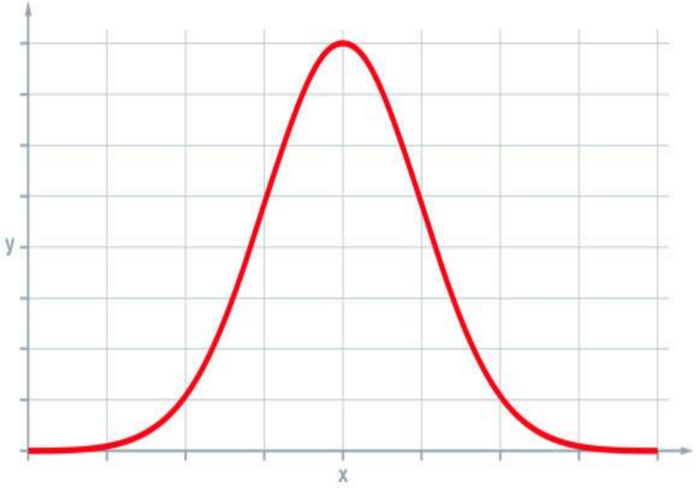
WHAT IS METROLOGY?

- The science of Metrology also includes uncertainties of measurements
  - If a device produces a 12 digit answer, how many digits are meaningful?
  - Methodology and mathematics have been developed to determine measurement *uncertainty* on a statistical basis



UNCERTAINTIES

- A *Standard* uncertainty is a one σ uncertainty and is designated by the lowercase "u"
- This has a 68% confidence interval
    - Multiply by 2 for 95%, 2.58 for 99% and 3 for 99.7%
- A metrologically sound measurement will be in this format: <value><units><uncertainty><confidence interval>
- A measurement may be a function of several variables, each with their own uncertainty



PROPAGATION OF UNCERTAINTIES

- The total squared uncertainty of a function of several variables, each with their own uncertainty and possibly correlated is

$$u_{total}^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j} u(x_i, x_j)$$

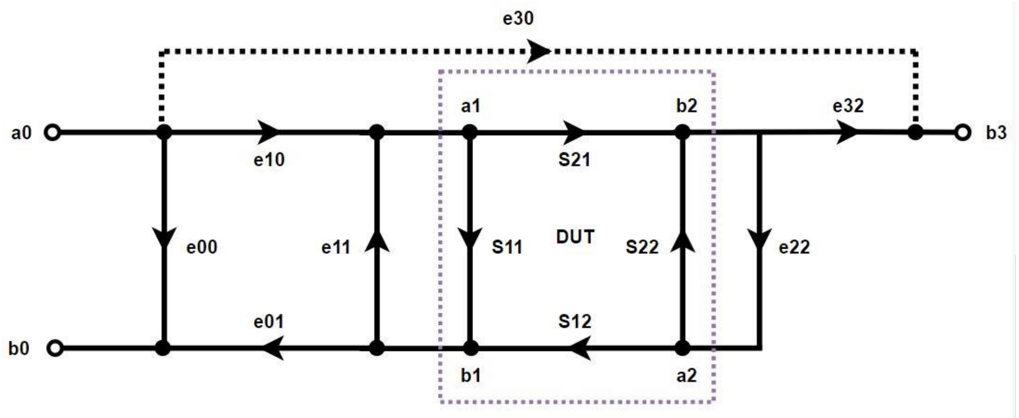
- Where the Measurement $y = f(x_1, x_2, \ldots\ x_N)$
- The partials are the *sensitivity factors* for the x_i
- The second term contains the cross-correlation uncertainties for correlated variables if any

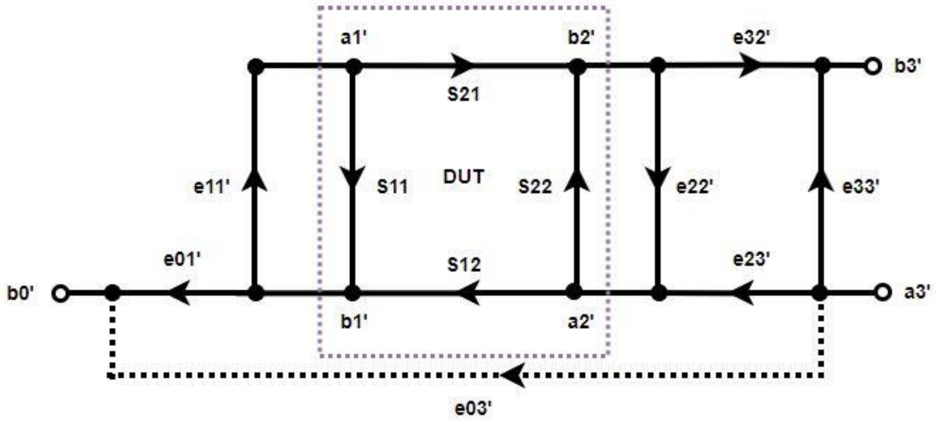
VNA MEASUREMENT ERRORS

- Prior to user calibration, a VNA measurement will contain errors, both systematic and random
  - *Systematic errors* may be corrected with calibration
  - *Random errors* cannot be corrected and may only be minimized through careful measurement technique
    - Random errors are generated by random processes such as thermal noise, connector loss and cable movement
- The following slides will focus on Systematic errors only

12-TERM ERROR MODEL

- The 12-Term error model is useful for modeling the systematic errors of a VNA measurement
- These error terms represent real physical processes which lead to an intuitive understanding
- The forward and reverse 12-Term Models are shown here





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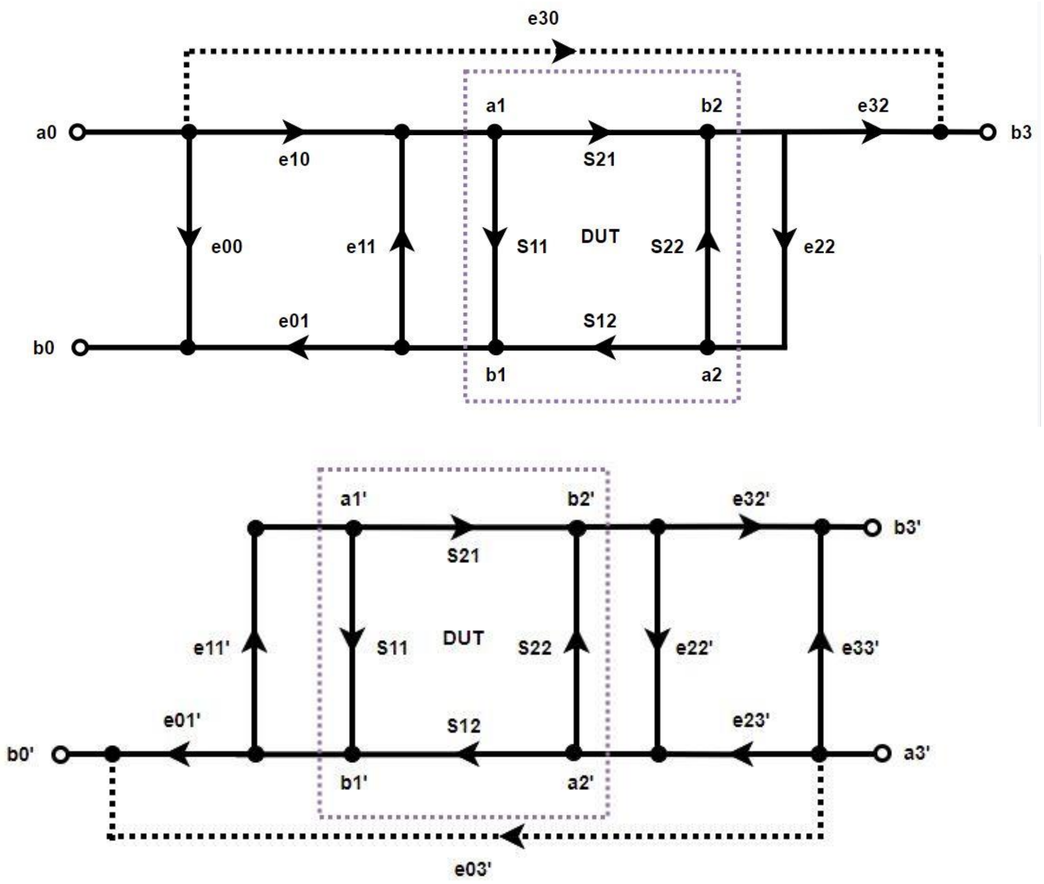
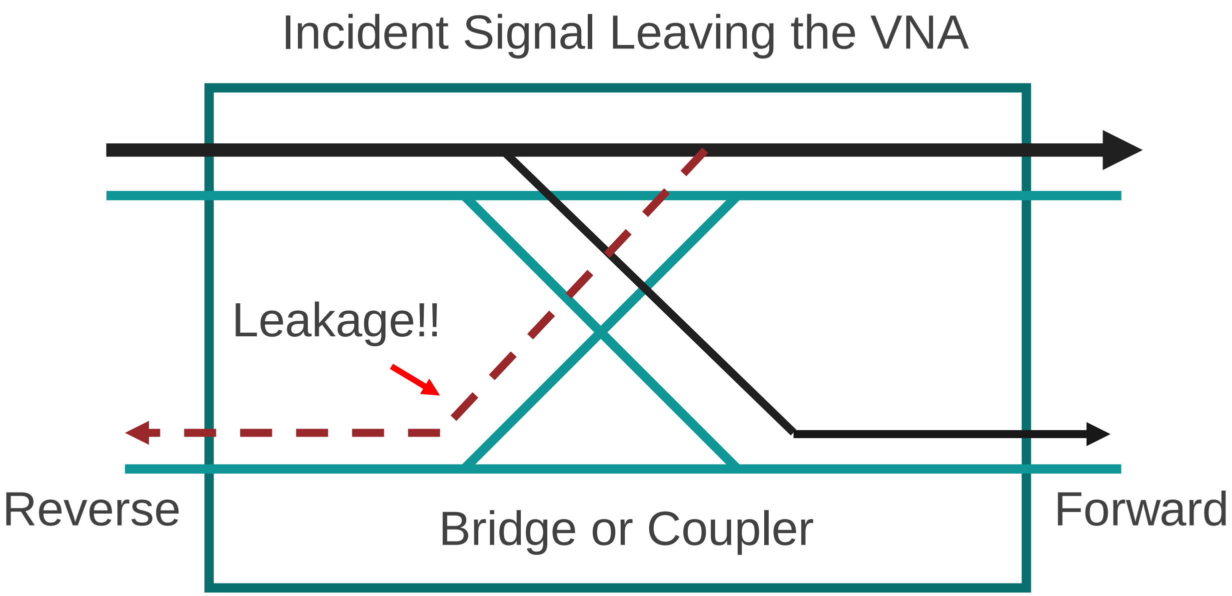
12-TERM ERROR MODEL

- The various errors are (Forward):
  - e_{00}, Directivity error
  - $e_{10}e_{01}$, Reflection Tracking error
  - e_{11}, Source Match error
  - e_{22}, Load Match error
  - e_{32}, Transmission Tracking error
  - e_{30}, Isolation error
- Errors in the Reverse direction are similar
- What are these errors?



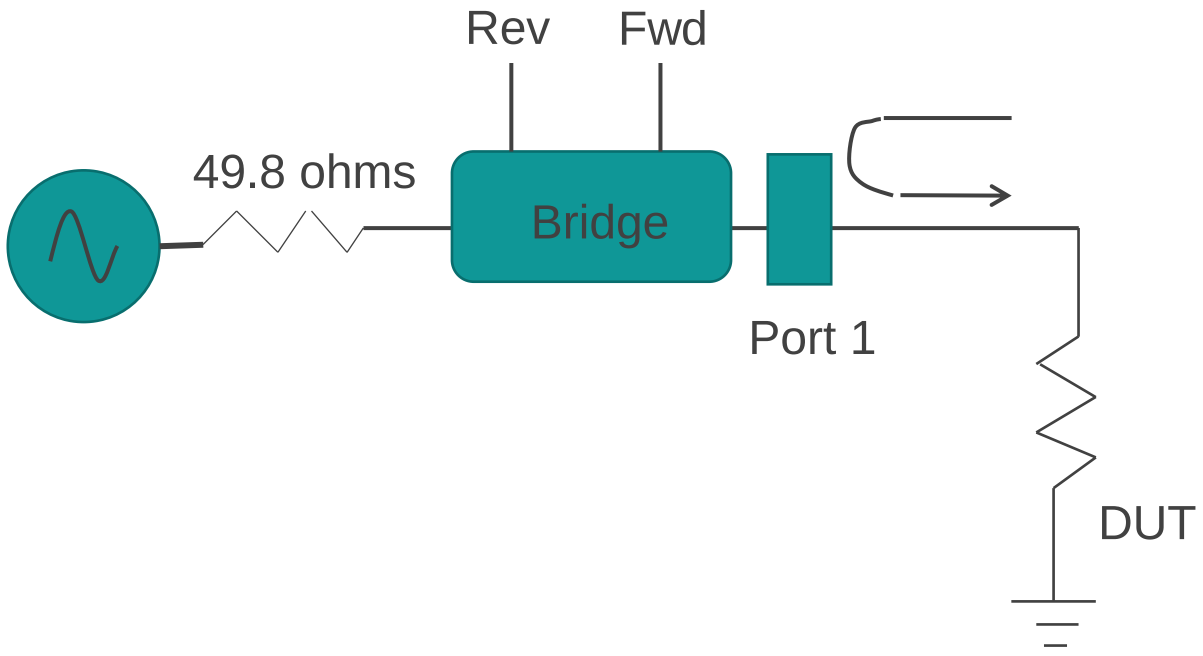
DIRECTIVITY ERROR (E00,E33’)

- In the VNA, A “Bridge” or Directional Coupler separates the forward (incident) from the reverse traveling waves (reflected)
- In an Ideal bridge the Forward Coupled port would only couple the Forward Traveling wave and the Reverse Coupled port would only couple the Reverse Traveling wave
- BUT, There will be some leakage from the forward traveling wave into the reverse coupled port
- This leakage is the Directivity Error



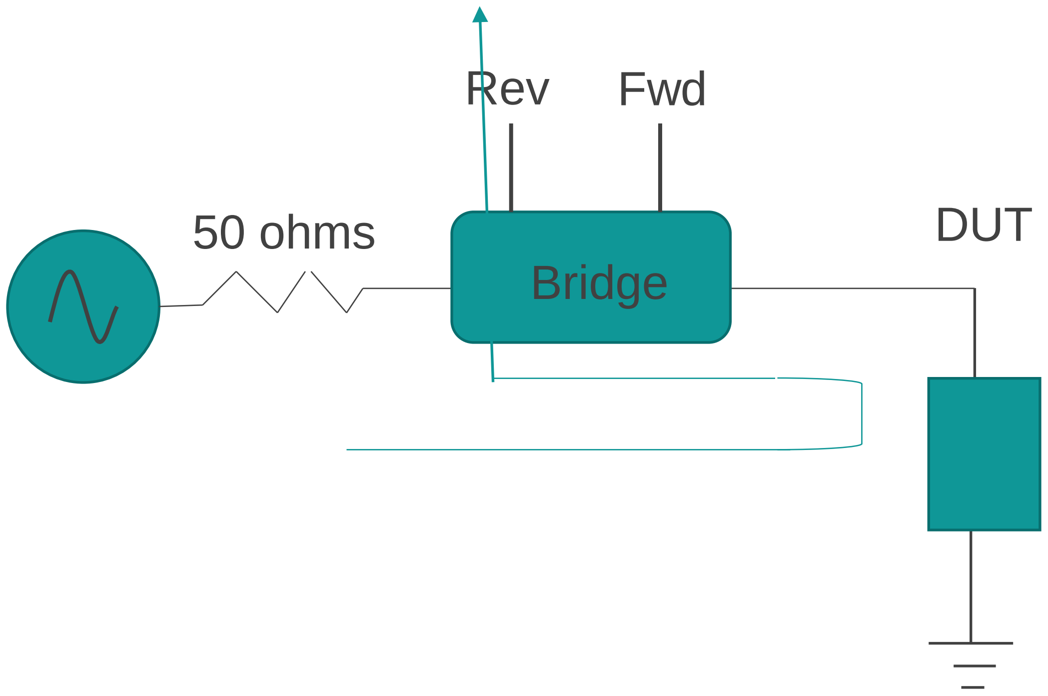
SOURCE MATCH ERROR (E11,E22')

- The output impedance of the VNA should be a perfect 50 ohm source, but it may not be perfect. The connector and the cable attached to Port-1 will alter the source impedance as well
- The reflected signal from the DUT sees an impedance mismatch at the reference plane and some signal is reflected back to the forward direction.
- This is called Source Match Error



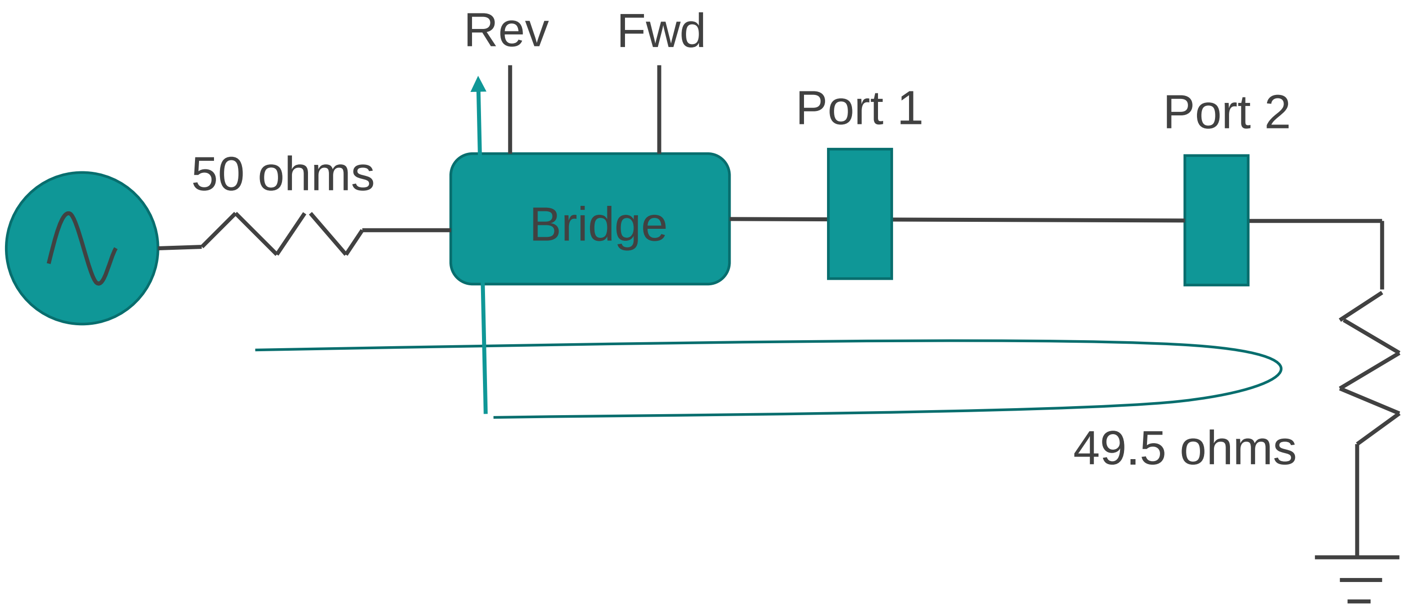
REFLECTION TRACKING ERROR (E01E10,E23'E32')

- The signal path from the Stimulus through the Bridge to the DUT and back through the Bridge to the Reverse port and then to mixers and amplifiers before digitization is not perfectly flat. There will be some frequency dependent attenuation.
- The difference between a perfectly flat response and the sloping response is called Reflection Tracking Error

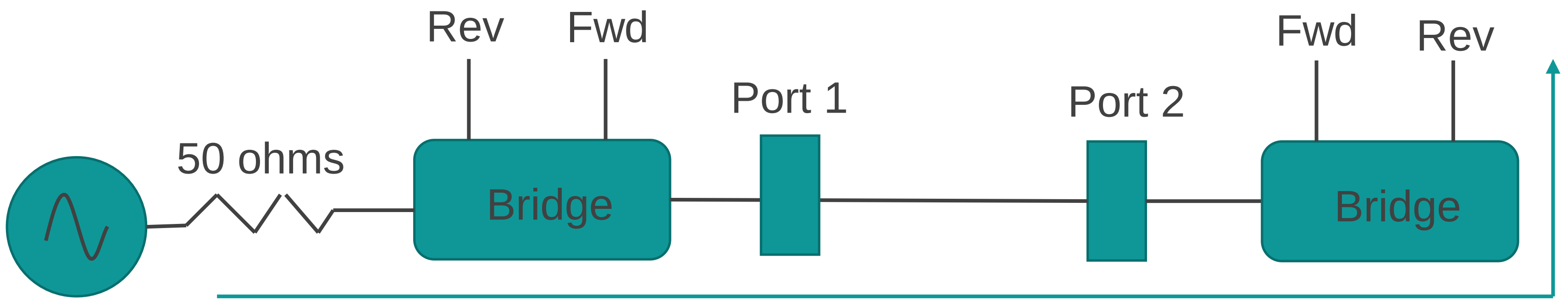


LOAD MATCH ERROR (E22,E11')

- When making a thru measurement between ports the impedance might not be precisely 50 ohms at the end of the Port 2 cable causing a reflection
- The Load Match is affected by the VNA input impedance and the Port 2 test cable and connectors
- This reflection is called Load Match Error

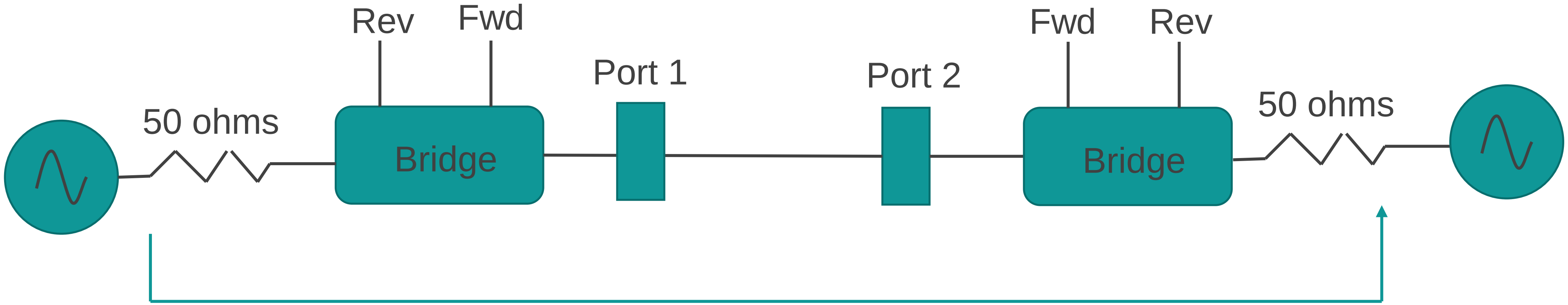


TRANSMISSION TRACKING ERROR (E312,E01')

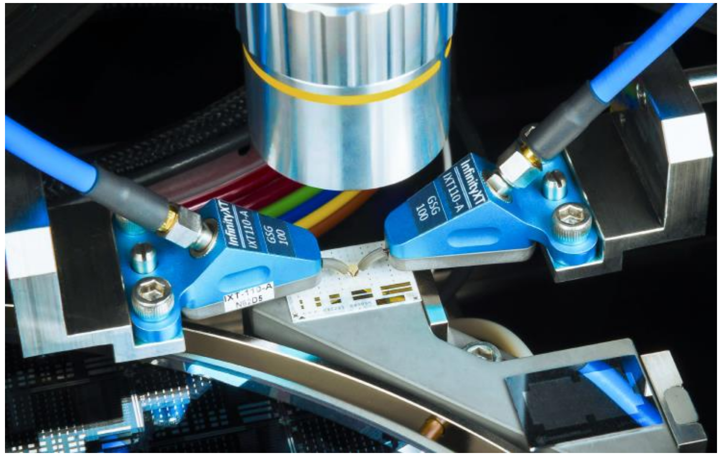


- When making a thru measurement between ports, the frequency response to the Port 2 receiver is not perfectly flat.
- It rolls off with frequency and may have some "bumps" in the response
- The Frequency response of the test cables contribute to this error
- The difference between a perfectly flat response and the measured value is called Transmission Tracking Error

ISOLATION ERROR (E30,E03')



- Within the VNA, there may be a small amount of leakage from port 1 to port 2. This is usually below the noise floor in a 10 Hz measurement and can therefore be ignored.
- There can be significant leakage between the probes of a wafer probe station so correction for isolation in this application is critical



FormFactor InfinityXt Probes shown here

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SOLVING FOR DUT S-PARAMETERS

- After full 2-port calibration, each S-parameter requires the measurement of all four raw S-parameters.
- That is why a 2-port analyzer sweep twice, once in each direction before displaying an S_{11} trace
- Port 2 is not assumed to be a perfect load. The Load Match corrections are required for a good S_{11} measurement

ERRORS AND RESIDUAL ERRORS

- Systematic errors are corrected by user calibration, but they don't go away entirely
- For Instance, The Directivity error is measured by terminating the VNA Port with a calibration load
- There should then be no reflection so the leakage in the bridge could be measured and subsequently removed from measurements
- But even the best calibration load has some reflection, perhaps -30 dB from 8-26.5 GHz for a 3.5 mm Load in a $7,000 calibration kit. (An ACM can provide **46 dB** uncertainty/Return Loss with its databased load)
- This reflection is seen at the reflection port of the bridge so the leakage may only be corrected down to this level thus setting a "floor" for reflection measurements
- Errors such as this which remain after calibration are called *Residual Errors.*
- After calibration, the error models remain the same but with the smaller residual errors replacing raw errors

ERRORS AND RESIDUAL ERRORS

- Using some algebra or network diagram theory, It’s not difficult to determine the actual S-Parameters of the DUT from the measured values with the error terms we’ve defined.
  - For Network diagram manipulation, see “One-Port VNA Calibration: A Look Under the Hood”, Brian Walker, Microwaves & RF, January 2021

$$S_{11} = \frac{\left(\frac{S_{11m}-e_{00}}{e_{10}-e_{01}}\right)\left[1+e'_{22}\left(\frac{S_{22m}-e'_{33}}{e'_{23}e'_{32}}\right)\right]-e_{22}\left(\frac{S_{21m}-e_{30}}{e_{10}e_{32}}\right)\left(\frac{S_{12m}-e'_{03}}{e'_{23}e'_{01}}\right)}{D}$$

$$S_{12} = \frac{\left(\frac{S_{12m}-e'_{03}}{e'_{23}-e'_{01}}\right)\left[1+(e_{11}-e'_{11})\left(\frac{S_{11m}-e_{00}}{e_{10}e_{01}}\right)\right]}{D}$$

$$S_{21} = \frac{\left(\frac{S_{21m}-e_{30}}{e_{10}-e_{32}}\right)\left[1+(e'_{22}-e_{22})\left(\frac{S_{22m}-e'_{33}}{e'_{23}e'_{32}}\right)\right]}{D}$$

$$S_{22} = \frac{\left(\frac{S_{22m}-e_{33}}{e'_{23}-e'_{32}}\right)\left[1+e_{11}\left(\frac{S_{11m}-e_{00}}{e_{10}e_{01}}\right)\right]-e'_{11}\left(\frac{S_{21m}-e_{30}}{e_{10}e_{32}}\right)\left(\frac{S_{12m}-e'_{03}}{e'_{23}e'_{01}}\right)}{D}$$

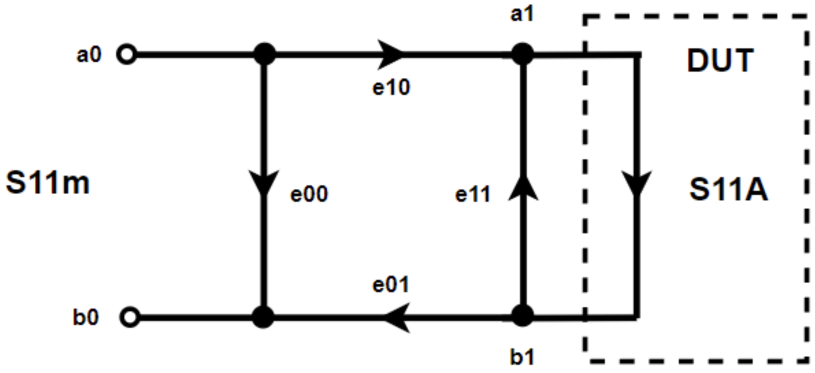
$$D = \left[1 + e_{11}\left(\frac{S_{11m}-e_{00}}{e_{10}e_{01}}\right)\right]\left[1 + e'_{22}\left(\frac{S_{22m}-e'_{33}}{e'_{23}e'_{32}}\right)\right] - e_{22}e'_{11}\left(\frac{S_{21m}-e_{30}}{e_{10}e_{32}}\right)\left(\frac{S_{12m}-e'_{03}}{e'_{23}e'_{01}}\right)$$

ERRORS AND RESIDUAL ERRORS

- These equations for a full 2-port measurement do not provide a very good intuitive understanding of the effects of the errors on the outcome
- The 1-Port calibration for an S11 measurement is simpler and would be similar to an S11 measurement after full 2-port calibration with a perfect load match.

$$S_{11a} = \frac{S_{11m} - e_{00}}{S_{11m}e_{11} - \Delta_e}$$

$$\Delta_e = e_{00}e_{11} - e_{01}e_{10}$$



- In principle, the Directivity error e_{00} dominates the error contributions

ESTIMATION OF RESIDUAL ERRORS

- Residual Errors are determined by the uncertainties of the Calibration Kit and may be estimated as follows.
- Greek letters will be used to avoid confusion since the equations may contain both raw and residual errors

$$\delta = -\mu_2 = -\Delta_L \qquad T_1 = \frac{\sqrt{\Delta_s^2 + \Delta_o^2}}{2} \qquad \mu_1 = \sqrt{\Delta_L^2 + \frac{\Delta_s^2 + \Delta_o^2}{4}} \cong \Delta_L \qquad T_2 = M_1\mu_2 + M_2\mu_1$$

- Where δ is Residual Directivity, μ_1 and μ_2 are Residual Source and Load Match
- M_1 and M_2 are Raw Source and Load Match errors
- T_1 and T_2 are Residual Reflection and Transmission Tracking
- $\Delta_L, \Delta_s, and\ \Delta_o$ are Calibration Load uncertainty and Calibration Short and Open phase uncertainty in radians
- All terms are in in linear format (not Log Magnitude)

ESTIMATION OF RESIDUAL ERRORS

- Note how the Residual Directivity is a direct function of the Calibration Load uncertainty (Return Loss)
- For small Open and Short phase error, the Residual Source Match is also approximately equal to the Calibration Load uncertainty
- Reflection Tracking error is very dependent on the phase uncertainty of the Calibration Open and Short standards
- The Raw Source and Load match have some effect on the Residual Transmission Tracking error

ESTIMATION OF RESIDUAL ERRORS

- As stated before, **Residual Errors are primarily determined by the uncertainties of the Calibration Kit**, with Raw Source and Load match having a small effect on Residual Transmission Tracking
- Other than this, the VNA itself has very little effect on Residual Error Parameters
- It can be said that a VNA with good Raw Directivity, Source and Load Match will exhibit somewhat better stability with changes in temperature
- A VNA with great Raw parameters cannot make up for a Calibration kit with poor uncertainties
- An Automatic Calibration Module should be used for calibration if precision results are desired

REFLECTION MEASUREMENT UNCERTAINTY

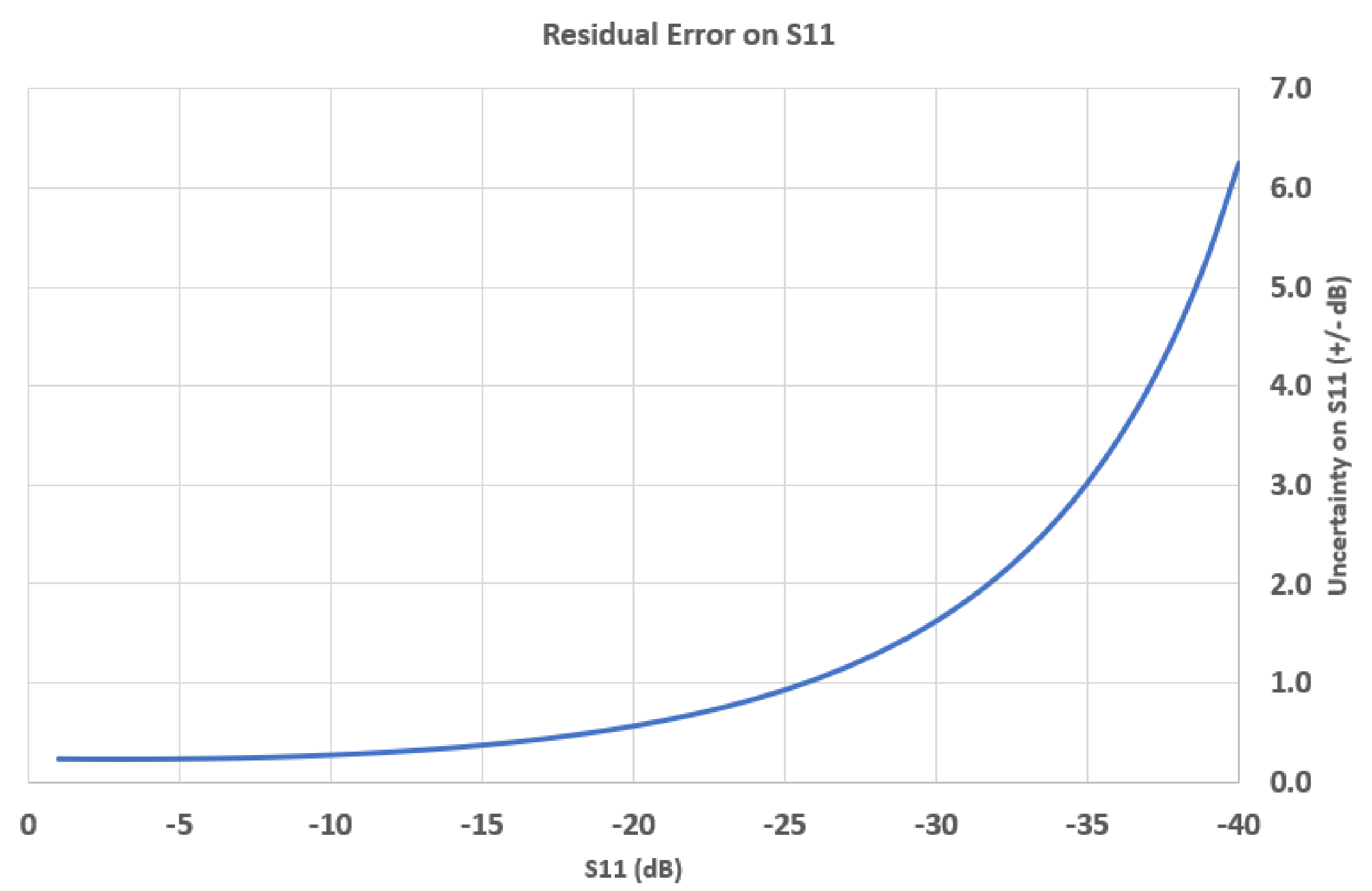
- The measurement uncertainties may be estimated from the Residual errors and Reflection values expressed in linear format
- For Reflection Measurements:

$$\Delta S_{ii} = \delta + T_1 * S_{ii} + \mu_1 * S_{ii}^2 + \mu_2 * S_{ji} * S_{ij} + R$$

- The term containing "μ_2" can be dropped for a 1-Port measurement of a DUT
- T1 here may include a small contribution for receiver non-linearity
- The Residual Directivity dominates for small reflections
- Reflection Tracking and Residual Source match dominate for high reflections
- Residual Source match contribution falls off quickly as reflection becomes smaller, leaving Tracking

REFLECTION MEASUREMENT UNCERTAINTY

- For δ = -46 dB, T_1=0.1 dB, μ_1= -40 dB, μ_2 is assumed to be a perfect match and R=0, the Reflection Uncertainty may be charted as shown here.
- Calibration with an Automatic Calibration Module (ACM) is required to attain δ = -46 dB
- The measurement uncertainty goes to infinity at -46 dB



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TRANSMISSION MEASUREMENT UNCERTAINTY

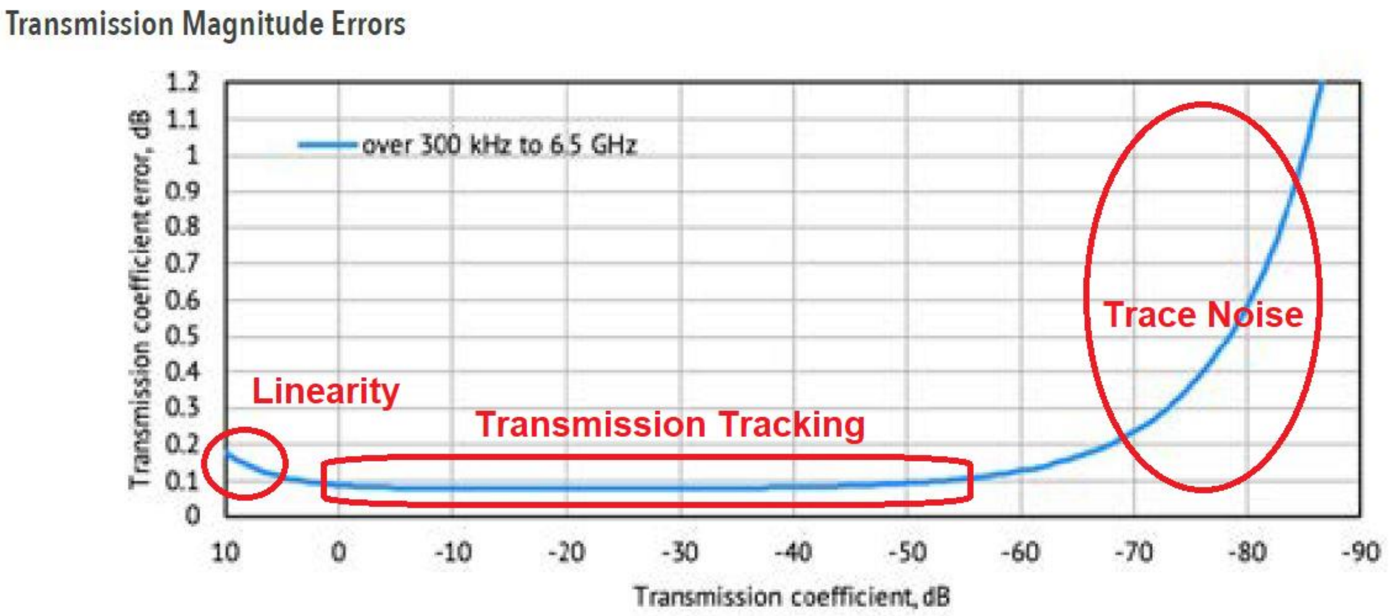
- The measurement uncertainties may be estimated from the Residual errors and Reflection values expressed in Logarithmic format
- For Transmission Measurements:

$$\Delta S_{ij} = \mathrm{T}_2 + M + I + R$$

- Where "T_2" is Residual Transmission Tracking in dB as calculated from Residual and Raw Source and Load Match as before with linearity error added
- "M" is the uncertainty due to mismatch which may be taken as 0 in most cases
- "I" is uncertainty due to Residual (or Raw if uncorrected) Isolation e_{03} or e_{30} from the 12-Term Error Model
- "R" Represents contributions from Random Factors such as Connector Uncertainty, Thermal Drift, Trace Noise, Cable Flexure and System Repeatability

TRANSMISSION MEASUREMENT UNCERTAINTY

- The effects of these errors are shown in the Transmission Uncertainty chart for the S5065 VNA shown below:



Specifications are based on matched DUT, and IF bandwidth of 10 Hz

QUESTIONS?

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